Review Exercises

1. $3e^{3x-7}$ **2.** $e^x \sec^2(e^x)$ **3.** $3\sin^2 x \cos x$

4. $-\cot x$ **5.** $2\sin(1-2t)$ **6.** $\frac{2}{t^2}\csc^2\frac{2}{t}$

7. $-\frac{\sin x}{2\sqrt{1+\cos x}}$

8. $\frac{3x+1}{\sqrt{2x+1}}$

9. $3\sec(1+3\theta)\tan(1+3\theta)$

10. $-4\theta\tan(3-\theta^2)\sec^2(3-\theta^2)$

11. $-5x^2\csc 5x\cot 5x + 2x\csc 5x$

12. $\frac{1}{2x}, x>0$ **13.** $\frac{e^x}{1+e^x}$

14. $-xe^{-x}+e^{-x}$ **15.** e

16. $\cot x$, where x is an interval of the form $(k\pi, (k+1)\pi)$, k even

17. $-\frac{1}{\cos^{-1}x\sqrt{1-x^2}}$

18. $\frac{2}{\theta\ln 2}$

19. $\frac{1}{(t-7)\ln 5}, t>7$

20. $-8^{-t}\ln 8$

21. $\frac{2(\ln x)x^{\ln x}}{x}$

22. $\frac{(2\cdot 2^x)[x^3\ln 2 + x\ln 2 + 1]}{(x^2+1)^{3/2}}$ or

$\frac{(2x)2^x}{\sqrt{x^2+1}}\left(\frac{1}{2}+\ln 2 - \frac{x}{x^2+1}\right)$

23. $\frac{e^{\tan^{-1}x}}{1+x^2}$

24. $-\frac{u}{\sqrt{u^2-u^4}} = -\frac{u}{|u|\sqrt{1-u^2}}$

25. $\frac{t}{|t|\sqrt{t^2-1}} + \sec^{-1}t - \frac{1}{2t}$

26. $-\frac{2+2t^2}{1+4t^2} + 2t\cot^{-1}2t$

27. $\cos^{-1}z$

28. $-\frac{1}{x} + \frac{\csc^{-1}\sqrt{x}}{\sqrt{x-1}}$

29. -1

30. $2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{\cos\theta-\sin\theta-1}{(1-\cos\theta)^2}\right)$

31. For all $x \neq 0$

32. For all real x

33. For all $x<1$

34. For all $x \neq 0$

35. $-\frac{y+2}{x+3}$

36. $-\frac{1}{3}(xy)^{-1/5}$

37. $-\frac{y}{x}$ or $-\frac{1}{x^2}$

38. $\frac{1}{2y(x+1)^2}$

39. $-\frac{2x}{y^5}$ **40.** $\frac{1+2xy^2}{x^4y^3}$

41. $-2\frac{(3y^2+1)^2\cos x + 12y\sin^2 x}{(3y^2+1)^3}$

42. $\frac{2}{3}x^{-4/3}y^{1/3} + \frac{2}{3}x^{-5/3}y^{2/3} = \frac{8}{3}x^{-5/3}y^{1/3}$

43. $32e^{\sqrt[8]{2x}}$

44. $y = 32\sin(\sqrt[8]{2x})$

45. **(a)** $y = \frac{2}{\sqrt{3}}x - \sqrt{3}$ **(b)** $y = -\frac{\sqrt{3}}{2}x + \frac{5\sqrt{3}}{2}$

46. **(a)** $y+\sqrt{3} = 8\left(x-\frac{\pi}{3}\right)$ **(b)** $y+\sqrt{3} = -\frac{1}{8}\left(x-\frac{\pi}{3}\right)$

47. **(a)** $y = -\frac{1}{4}x + \frac{9}{4}$ **(b)** $y = 4x-2$

48. **(a)** $y = -\frac{5}{4}x + 6$ **(b)** $y = \frac{4}{5}x - \frac{11}{5}$

49. $y = x - 2\sqrt{2}$

50. $y = \frac{4}{3}x + 4\sqrt{2}$

51. $y = \frac{10}{3}x - 5\sqrt{3}$

52. $y = (1+\sqrt{2})x - \sqrt{2} - 1 - \frac{\pi}{4}$ or $y \approx 2.414x - 3.200$

53. **(a)** $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(\sin ax + b\cos x) = b$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(5x+3) = 3$. Thus $\lim_{x\to 0} f(x) = f(0) = 3$ if and only if $b = 3$

(b) $f'(x) = \begin{cases} a\cos ax - b\sin x, & x<0 \\ 5, & x>0 \end{cases}$. The slopes match at $x=0$ if and only if $a=5$.

(c) No. Although the slopes match, the function is not continuous.

54. **(a)** The function is continuous for all values of m, because the right-hand limit as $x \to 0$ is equal to $f(0) = 0$ for any value of m.

(b) The left-hand derivative at $x=0$ is 2, and the right-hand derivative at $x=0$ is m, so in order for the function to be differentiable at $x=0$, m must be 2.

55. **(a)** For all $x \neq 1$ **(b)** At $x=1$ **(c)** Nowhere

56. **(a)** For all x **(b)** Nowhere **(c)** Nowhere

57. **(a)** $[-1,1)\cup(1,3]$ **(b)** At $x=1$ **(c)** Nowhere

58. **(a)** $[-3,3]$ **(b)** Nowhere **(c)** Nowhere

59. **(a)** $-\cos x$ **(b)** $14x-13$ **(c)** $2t-3$ **(d)** $2t-30t^5$

60. **(a)** $\frac{2}{2x+7} - \frac{3}{3x+2}$ **(b)** 1

(c) $-2t$ **(d)** $\frac{80}{3\sqrt[6]{t}}$

61. **(a)** $y-4 = -\frac{1}{3}(x-2)$

(b) $y-2 = \frac{1}{3}(x-4)$

(c) $y-2 = \frac{1}{2}(x-2)$

62. **(a)** $y-3 = \frac{1}{2}(x-1)$

(b) $y-1 = -\frac{1}{2}(x-3)$

(c) $y-3 = -4(x-1)$

63. $\frac{dy}{dx} = \frac{(x+2)^5(2x-3)^4}{(x+17)^2}\left(\frac{5}{x+2} + \frac{8}{2x+3} - \frac{2}{x+17}\right)$

64. $\frac{dy}{dx} = (x^2+2)^{x+5}\left(\ln(x^2+2) + \frac{2x^2+10x}{x^2+2}\right)$

65. (a) $f(x) = \frac{x^2}{2}$ or $f(x) = \frac{x^2}{2} + C$
(b) $f(x) = e^x$ or $f(x) = Ce^x$
(c) $f(x) = e^{-x}$ or $f(x) = Ce^{-x}$
(d) $f(x) = e^x$ or $f(x) = e^{-x}$ or $f(x) = Ce^x + De^{-x}$
(e) $f(x) = \sin x$ or $f(x) = \cos x$ or $f(x) = C\sin x + D\cos x$

66. (a) $-13/10$ **(b)** $-1/3$
(c) $1/10$ **(d)** -1
(e) $-2/3$ **(f)** -12

67. (a) 5 **(b)** 0
(c) 8 **(d)** 2
(e) 4 **(f)** -1

68. $\sqrt{3}$ **69.** $-1/6$

70. (a) One possible answer:
$x(t) = 10\cos\left(t + \frac{\pi}{4}\right)$
$y(t) = 0$
(b) $5\sqrt{2}$
(c) $s = -10$ and $s = 10$
(d) At $t = \pi/4$:
Velocity $= -10$
Speed $= 10$
Acceleration $= 0$

71. (a) $A(-1, 1)$; $B(1, -1)$
(b) $C(-0.5, 2)$; $D(0.5, -2)$

72. (a) $A(-\sqrt{2}, -2\sqrt{2})$; $B(\sqrt{2}, 2\sqrt{2})$
(b) $C(-2, -2)$; $D(2, 2)$

73. (a) $A(-2, -2)$
(b) $B(-1, -3)$

74. At y-intercept $(0, 2\sqrt{2})$ the slope is $\frac{2+\sqrt{2}}{2}$.
At y-intercept $(0, -2\sqrt{2})$ the slope is $\frac{2-\sqrt{2}}{2}$.
At x-intercept $(2 + 2\sqrt{3}, 0)$ the slope is $\frac{\sqrt{3}}{\sqrt{3}+1}$.
At x-intercept $(2 - 2\sqrt{3}, 0)$ the slope is $\frac{\sqrt{3}}{\sqrt{3}-1}$.

75. 6

76. Every sinusoid with amplitude A and period p is the graph of some equation of the form $y = A\sin\left(\frac{2\pi}{p}x + k\right) + D$. The slope at any x is $\frac{dy}{dx} = A\cdot\frac{2\pi}{p}\cos\left(\frac{2\pi}{p}x + k\right)$. Since the maximum value of cosine is 1, the maximum slope is $\frac{2\pi A}{p}$.

77. Yes

78. (a) $P(0) \approx 1.339$, so initially, one student was infected
(b) 200
(c) After 5 days, when the rate is 50 students/day

79. (a) $-\frac{2}{3}$
(b) $-\frac{5}{27}$

80. $-1/(3\sqrt{3})$

81. (a) $g'(x) = k\cdot e^{kx} + f'(x)$, so $g'(0) = k + 3$.
$g''(x) = k^2\cdot e^{kx} + f''(x)$, so $g''(0) = k^2 - 1$.
(b) $h'(x) = b\sin(bx)f(x) + f'(x)\cos(bx)$, so $h'(0) = b\cdot\sin(0) + 3\cdot\cos(0) = 3$. Note that $h(0) = \cos(0)\cdot f(0) = 1\cdot 2 = 2$, so the tangent line has equation $y - 2 = 3(x - 0)$.

82. (a) $\frac{dy}{dx} = \frac{e^x - e^{-x}}{2}$
(b) $\frac{d^2y}{dx^2} = \frac{e^x + e^{-x}}{2}$
(c) At $x = 1$, $\frac{dy}{dx} = \frac{e^1 - e^{-1}}{2} = 1.175$ and $y = \frac{e^1 + e^{-1}}{2} = 1.543$. The tangent line has equation $y - 1.543 = 1.175(x - 1)$.
(d) The normal line has equation $y - 1.543 = -0.851(x - 1)$.
(e) The tangent line is horizontal where $dy/dx = 0$; that is, where $e^x = e^{-x}$. This is true only at $x = 0$.

83. (a) The domain of f is the interval $(-1, 1)$.
(b) $f'(x) = \frac{-2x}{1-x^2} = \frac{2x}{x^2-1}$ on the domain $(-1, 1)$.
(c) The domain of f' is the interval $(-1, 1)$.
(d) $f''(x) = \left(\frac{2x}{x^2-1}\right)' = \frac{2(x^2-1) - 2x(2x)}{(x^2-1)^2} = \frac{-2(x^2+1)}{(x^2-1)^2} < 0$
for all x in the domain of f, since $\frac{x^2+1}{(x^2-1)^2} > 0$ for all x between -1 and 1.

CHAPTER 5

Section 5.1

Quick Review 5.1

1. $\frac{-1}{2\sqrt{4-x}}$ **3.** $\frac{\sin(\ln x)}{x}$

5. (c) **7.** (d) **9.** ∞

11. (a) 1 **(b)** 1 **(c)** Undefined

Exercises 5.1

1. Minima at $(-2, 0)$ and $(2, 0)$, maximum at $(0, 2)$
3. Maximum at $(0, 5)$
5. Maximum at $x = b$, minimum at $x = c_2$; Extreme Value Theorem applies, so both the max and min exist.
7. Maximum at $x = c$, no minimum; Extreme Value Theorem doesn't apply, since the function isn't defined on a closed interval.
9. Maximum at $x = c$, minimum at $x = a$; Extreme Value Theorem doesn't apply, since the function isn't continuous.
11. Maximum value is $\frac{1}{4} + \ln 4$ at $x = 4$; minimum value is 1 at $x = 1$; local maximum at $\left(\frac{1}{2}, 2 - \ln 2\right)$
13. Maximum value is $\ln 4$ at $x = 3$; minimum value is 0 at $x = 0$.
15. Maximum value is 1 at $x = \frac{\pi}{4}$; minimum value is -1 at $x = \frac{5\pi}{4}$; local minimum at $\left(0, \frac{1}{\sqrt{2}}\right)$; local maximum at $\left(\frac{7\pi}{4}, 0\right)$
17. Maximum value is $3^{2/5}$ at $x = -3$; minimum value is 0 at $x = 0$. [(0, 0) is not a stationary point]
19. Min value 1 at $x = 2$